SOUTHERN LEHIGH SCHOOL DISTRICT
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Scope and Sequence for **Calculus**

Standards for Mathematical Practice:	
MP1 Make sense of problems and persevere in solving them. **MP2** Reason abstractly and quantitatively. **MP3** Construct viable arguments and critique the reasoning of others. **MP4** Model with mathematics.	**MP5** Use appropriate tools strategically. **MP6** Attend to precision. **MP7** Look for and make use of structure. **MP8** Look for and express regularity in repeated reasoning.

1. Limits

College Board Learning Objectives	College Board Essential Knowledge
EU 1.1: The concept of a limit can be used to understand the behavior of functions. **LO 1.1A(a):** Express limits symbolically using correct notation. **LO 1.1A(b):** Interpret limits expressed symbolically.	**EK1.1A1:** Given a function f, the limit of as $f(x)$ as x approaches c is a real number $\mathbb{R}$ if $f(x)$ can be made arbitrarily close to $\mathbb{R}$ by taking x sufficiently close to c (but not equal to c). If the limit exists and is a real number, then the common notation is $\lim_{x\to c} f(x) = \mathbb{R}$ **EK 1.1A2:** The concept of a limit can be extended to include one-sided limits, limits at infinity, and infinite limits. **EK 1.1A3:** A limit might not exist for some functions at particular values of x Some ways that the limit might not exist are if the function is unbounded, if the function is oscillating near this value, or if the limit from the left does not equal the limit from the right.
LO 1.1B: Estimate limits of functions.	**EK 1.1B1:** Numerical and graphical information can be used to estimate limits.

1. Limits – *Continued…*

College Board Learning Objectives	College Board Essential Knowledge
LO 1.1C: Determine limits of functions.	**EK 1.1C1:** Limits of sums, differences, products, quotients, and composite functions can be found using the basic theorems of limits and algebraic rules. **EK 1.1C2:** The limit of a function may be found by using algebraic manipulation, alternate forms of trigonometric functions, or the squeeze theorem.
LO 1.1D: Deduce and interpret behavior of functions using limits.	**EK 1.1D1:** Asymptotic and unbounded behavior of functions can be explained and described using limits
EU 1.2: Continuity is a key property of functions that is defined using limits **LO 1.2A:** Analyze functions for intervals of continuity or points of discontinuity	**EK 1.2A1:** A function f is continuous at $x = c$ provided that $f(c)$ exists, $\lim_{x \to c} f(x)$ exists, and $\lim_{x \to c} f(x) = f(c)$. **EK 1.2A2:** Polynomial, rational, power, exponential, logarithmic, and trigonometric functions are continuous at all points in their domains. **EK 1.2A3:** Types of discontinuities include removable discontinuities, jump discontinuities, and discontinuities due to vertical asymptotes.

2. Derivatives

College Board Learning Objectives	College Board Essential Knowledge
EU 2.1: The derivative of a function is defined as the limit of a difference quotient and can be determined using a variety of strategies. **LO 2.1A:** Identify the derivative of a function as the limit of a difference quotient.	**EK 2.1A1:** The difference quotients $\frac{f(a+h)-f(a)}{h}$ and $\frac{f(x)-f(a)}{x-a}$ express the average rate of change a function over an interval. **EK 2.1A2:** The instantaneous rate of change of a function at a point can be expressed by $\lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$ or $\lim_{h\to 0}\frac{f(x)-f(a)}{x-a}$, provided that the limit exists. These are common forms of the definition of the derivative and are denoted $f'(a)$. **EK 2.1A3:** The derivative of f is the function whose value at x is $\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ provided this limit exists. **EK 2.1A4:** For $y = f(x)$, notations for the derivative include $\frac{dy}{dx}$, $f'(x)$, and y'. **EK 2.1A5:** The derivative can be represented graphically, numerically, analytically, and verbally.
LO 2.1C: Calculate derivatives.	**EK 2.1C1:** Direct application of the definition of the derivative can be used to find the derivative for selected functions, including polynomial, power, sine, cosine, exponential, and logarithmic functions. **EK 2.1C2:** Specific rules can be used to calculate derivatives for classes of functions, including polynomial, rational, power, exponential, logarithmic, trigonometric, and inverse trigonometric. **EK 2.1C3:** Sums, differences, products, and quotients of functions can be differentiated using derivative rules. **EK 2.1C4:** The chain rule provides a way to differentiate composite functions. **EK 2.1C5:** The chain rule is the basis for implicit differentiation.

2. Derivatives – *Continued…*

College Board Learning Objectives	College Board Essential Knowledge
LO 2.1D: Determine higher order derivatives.	**EK 2.1D1:** Differentiating f' produces the second derivative f'', provided the derivative of f' exists; repeating this process produces higher order derivatives of f. **EK 2.1D2:** Higher order derivatives are represented with a variety of notations. For $y = f(x)$, notations for the second derivative include $\frac{d^2y}{dx^2}$, $f''(x)$, and y''. Higher order derivatives can be denoted $\frac{d^ny}{dx^n}$ or $f^{(n)}(x)$.
EU 2.2: A function's derivative, which is itself a function, can be used to understand the behavior of the function. **LO 2.2A:** Use derivatives to analyze properties of a function	**EK 2.2A1:** First and second derivatives of a function can provide information about the function and its graph including intervals of increase or decrease, local (relative) and global (absolute) extrema, intervals of upward or downward concavity, and points of inflection. **EK 2.2A2:** Key features of functions and their derivatives can be identified and related to their graphical, numerical, and analytical representations. **EK 2.2A3:** Key features of the graphs of f, f', and f'' are related to one another.
LO 2.2B: Recognize the connection between differentiability and continuity.	**EK 2.2B1:** A continuous function may fail to be differentiable at a point in its domain. **EK 2.2B2:** If a function is differentiable at a point, then it is continuous at that point.
EU 2.3: The derivative has multiple interpretations and applications including those that involve instantaneous rates of change. **LO 2.3A:** Interpret the meaning of a derivative within a problem.	**EK 2.3A1:** The unit for $f'(x)$ is the unit for f divided by the unit for *x*. **EK 2.3A2:** The derivative of a function can be interpreted as the instantaneous rate of change with respect to its independent variable.
LO 2.3B: Solve problems involving the slope of a tangent line.	**EK 2.3B1:** The derivative at a point is the slope of the line tangent to a graph at that point on the graph.
LO 2.3C: Solve problems involving related rates, optimization, rectilinear motion	**EK 2.3C1:** The derivative can be used to solve rectilinear motion problems involving position, speed, velocity, and acceleration. **EK 2.3C2:** The derivative can be used to solve related rates problems, that is, finding a rate at which one quantity is changing by relating it to other quantities whose rates of change are known. **EK 2.3C3:** The derivative can be used to solve optimization problems, that is, finding a maximum or minimum value of a function over a given interval.
LO 2.3D: Solve problems involving rates of change in applied contexts.	**EK 2.3D1:** The derivative can be used to express information about rates of change in applied contexts.
LO 2.3E: Verify solutions to differential equations.	**EK 2.3E1:** Solutions to differential equations are functions or families of functions. **EK 2.3E2:** Derivatives can be used to verify that a function is a solution to a given differential equation.

3. Integrals and the Fundamental Theorem of Calculus

College Board Learning Objectives	College Board Essential Knowledge
EU 3.1: Antidifferentiation is the inverse process of differentiation. **LO 3.1A:** Recognize antiderivatives of basic functions.	**EK 3.1A1:** An antiderivative of a function f is a function g whose derivative is f. **EK 3.1A2:** Differentiation rules provide the foundation for finding antiderivatives.
EU 3.2: The definite integral of a function over an interval is the limit of a Riemann sum over that interval and can be calculated using a variety of strategies. **LO 3.2A(a):** Interpret the definite integral as the limit of a Riemann sum. **LO 3.2A(b):** Express the limit of a Riemann sum in integral notation.	**EK 3.2A1:** A Riemann sum, which requires a partition of an interval *I*, is the sum of products, each of which is the value of the function at a point in a subinterval multiplied by the length of that subinterval of the partition. **EK 3.2A2:** The definite integral of a continuous function f over the interval $[a, b]$, denoted by $\int_a^b f(x)\,dx$, is the limit of Riemann sums as the widths of the subintervals approach 0. That is, $\int_a^b f(x)\,dx = \lim_{\max \Delta x_i \to 0} \sum_{i=1}^{n} f(x_i^*)\Delta x_i$, where x_i^* is a value in the *i*th subinterval, Δx_i is the width of the *i*th subinterval, n is the number of subintervals, and $\max \Delta x_i$ is the width of the largest subinterval. Another form of the definition is $\int_a^b f(x)\,dx = \lim_{n \to \infty} \sum_{i=1}^{n} f(x_i^*)\Delta x_i$, where $\Delta x_i = \frac{b-a}{n}$ and x_i^* is a value in the *i*th subinterval. **EK 3.2A3:** The information in a definite integral can be translated into the limit of a related Riemann sum, and the limit of a Riemann sum can be written as a definite integral.
LO 3.2B: Approximate a definite integral	**EK 3.2B1:** Definite integrals can be approximated for functions that are represented graphically, numerically, algebraically, and verbally. **EK 3.2B2:** Definite integrals can be approximated using a left Riemann sum, a right Riemann sum, a midpoint Riemann sum, or a trapezoidal sum; approximations can be computed using either uniform or nonuniform partitions.
LO 3.2C: Calculate a definite integral using areas and properties of definite integrals.	**EK 3.2C1:** In some cases, a definite integral can be evaluated by using geometry and the connection between the definite integral and area. **EK 3.2C2:** Properties of definite integrals include the integral of a constant times a function, the integral of the sum of two functions, reversal of limits of integration, and the integral of a function over adjacent intervals.

3. Integrals and the Fundamental Theorem of Calculus – *Continued…*

College Board Learning Objectives	College Board Essential Knowledge
LO 3.3B(a): Calculate antiderivatives. **LO 3.3B(b):** Evaluate definite integrals.	**EK 3.3B1:** The function defined by $F(x) = \int_a^x f(t)\,dt$ is an antiderivative of f. **EK 3.3B2:** If f is continuous on the interval $[a, b]$ and F is an antiderivative of f, then $\int_a^b f(x)\,dx = F(b) - F(a)$. **EK 3.3B3:** The notation $\int f(x)\,dx = F(x) + C$ means that $F'(x) = f(x)$, and $\int f(x)\,dx$ is called an indefinite integral of the function f. **EK 3.3B5:** Techniques for finding antiderivatives include algebraic manipulation such as long division and completing the square, substitution of variables.
EU 3.4: The definite integral of a function over an interval is a mathematical tool with many interpretations and applications involving accumulation **LO 3.4A**: Interpret the meaning of a definite integral within a problem	**EK 3.4A1**: A function defined as an integral represents an accumulation of a rate of change. **EK 3.4A2:** The definite integral of the rate of change of a quantity over an interval gives the net change of that quantity over that interval. **EK 3.4A3:** The limit of an approximating Riemann sum can be interpreted as a definite integral.
LO 3.4B: Apply definite integrals to problems involving the average value of a function.	**EK 3.4B1:** The average value of a function f over an interval $[a, b]$ is $\frac{1}{b-a}\int_a^b f(x)\,dx$.
LO 3.4C: Apply definite integrals to problems involving motion.	**EK 3.4C1:** For a particle in rectilinear motion over an interval of time, the definite integral of velocity represents the particle's displacement over the interval of time, and the definite integral of speed represents the particle's total distance traveled over the interval of time.
LO 3.4D: Apply definite integrals to problems involving area, volume	**EK 3.4D1:** Areas of certain regions in the plane can be calculated with definite integrals **EK 3.4D2:** Volumes of solids with known cross sections, including discs and washers, can be calculated with definite integrals.
LO 3.4E: Use the definite integral to solve problems in various contexts.	**EK 3.4E1:** The definite integral can be used to express information about accumulation and net change in many applied contexts.
EU 3.5: Antidifferentiation is an underlying concept involved in solving separable differential equations. Solving separable differential equations involves determining a function or relation given its rate of change **LO 3.5A:** Analyze differential equations to obtain general and specific solutions.	**EK 3.5A1**: Antidifferentiation can be used to find specific solutions to differential equations with given initial conditions, including applications to motion along a line. **EK 3.5A2:** Some differential equations can be solved by separation of variables.